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Minisurf – A minimal surface generator for finite element modeling and additive manufacturing

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ABSTRACT

Triply periodic minimal surfaces (TPMSs) have long been studied by mathematicians but have recently garnered significant interest from the engineering community as ideal topologies for shell-based architected materials with both mechanical and functional applications. Here, we present a TPMS generator, *MiniSurf*. It combines surface visualization and CAD file generation (for both finite element modeling and additive manufacturing) within one single GUI. *MiniSurf* presently can generate 19 built-in and one user-defined triply periodic minimal surfaces based on their level-set surface approximations. Users can fully control the periodicity and precision of the generated surfaces. We show that *MiniSurf* can potentially be a very useful tool in designing and fabricating architected materials.

Code metadata

Current Code version	*v1.0*
Permanent link to code/repository used for this code version	https://github.com/SoftwareImpacts/SIMPAC-2020-28
Permanent link to reproducible capsule	https://codeocean.com/capsule/1851964/tree/v1
Legal Software License	*MIT*
Code versioning system used	*None*
Software code languages, tools, and services used	*Matlab*
Compilation requirements, operating environments, & dependencies	
If available Link to developer documentation/manual	
Support email for questions	mengtinh@uci.edu and Valdevit@uci.edu

Software metadata

Current software version	*v1.0*
Permanent link to executables of this version	https://github.com/mengtinh/MiniSurf
Permanent link to Reproducible Capsule	https://codeocean.com/capsule/1851964/tree/v1
Legal Software License	*MIT*
Computing platform / Operating System	*Microsoft Windows*
Installation requirements & dependencies	*Matlab Runtime*
If available Link to user manual — if formally published include a reference to the publication in the reference list	https://github.com/mengtinh/MiniSurf/blob/master/User%20Manual.pdf
Support email for questions	mengtinh@uci.edu and Valdevit@uci.edu

1. Introduction

For decades, scientists and engineers have been striving to design and fabricate new multiphase materials with controlled phase topologies – often termed “architected materials” or “metamaterials” – with unprecedented and tunable combinations of properties; architected cellular materials, where one phase is void, are the most notable examples. In terms of mechanical behavior, significant efforts have focused on designing architected materials that are stiff, strong and

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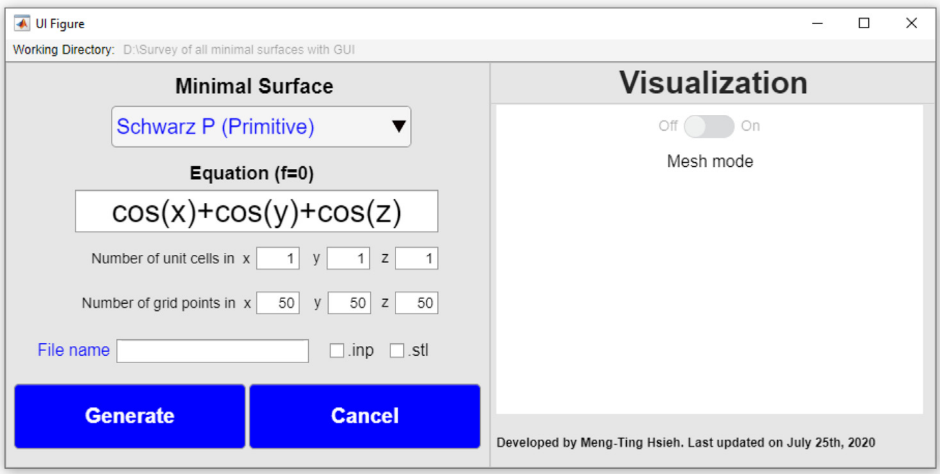

Fig. 1. Display of *MiniSurf* GUI: Control panel is on the left and visualization panel is on the right.

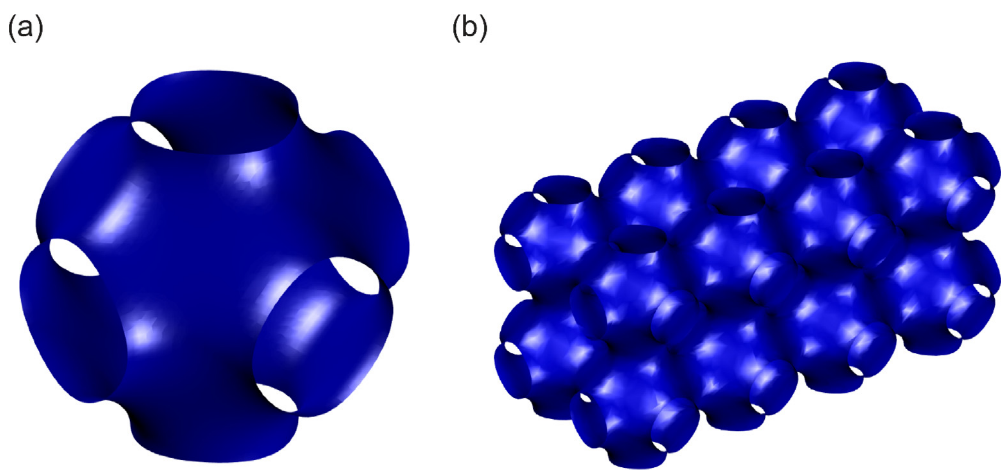

Fig. 2. Display of (a) single unit cell and (b) 4 × 2 × 2 unit cells of Schwarz P surface.

tough at very low density, by optimizing the topology of the material phases. Traditionally, topologies have largely been limited to beam-based structures, such as honeycombs in 2D [1–7] and octet lattices in 3D [8–14]. More recently, interest has shifted to shell-based topologies with minimal surface characteristics, such as triply periodic minimal surfaces (TPMS) [15–20] and isotropic stochastic spinodal minimal surfaces [21–23]; while more challenging to fabricate, these topologies are devoid of nodes and other stress intensification regions, which results in improved strength and toughness [21,24–26] as well as efficient fluid transport at low pressure drops [27–29]. Many studies of these minimal surface topologies have been motivated by the development of superior additive manufacturing (AM) technologies that enable their fabrication, and generally employ finite element modeling (FEM) for calculation of their mechanical and functional response; as a consequence, there is an increasing need for quick and accurate generation of computer-aided design (CAD) files for periodic cellular materials based on TPMS topologies, to be employed both for numerical analysis and additive manufacturing.

In this article, we present an efficient software application called "*MiniSurf*", which combines surface visualization and CAD file generation (for both FEM and AM) within one single graphical user interface (GUI). We briefly describe and illustrate the main software features. In addition, we highlight the impact of this package on current and potential applications in the field of architected materials design. Finally, we discuss the software limitations and future improvements.

2. Description and features

Minisurf is a software package that runs on Matlab Runtime (a freely accessible Matlab compiler) for visualization and generation of triply periodic minimal surface CAD files (with .inp extension for FEM through Simulia Abaqus and/or .stl extension for AM). The software package has a sleek and simple GUI consisting of two panels: a control panel (left) and a visualization panel (right), as shown in Fig. 1. The control panel allows users to select from the built-in library of minimal surfaces, as well as to type in the custom level-set equation of any desired surface. To facilitate generation of periodic architected materials, users can adjust the number of unit cells N_i, with $i = x, y, z$, along the x, y and z-directions, to produce specimens of different aspect ratios and number of unit cells, as shown in Fig. 2(a) and (b). In addition, the precision of the generated surfaces, governed by number of composing facets, can be fine-tuned by changing the number of mesh grid points P_i (with $i = x, y, z$) along the x, y, and z-directions. The generated minimal surfaces will be shown in the visualization panel in either non-mesh or mesh mode, as illustrated in Fig. 3(a) and (b).

MiniSurf currently has 19 built-in minimal surfaces. All these minimal surfaces are generated by meshing their implicit level-set approximations $f(x, y, z) = c$, where c is a constant and x, y, and z represent the location of $P_x \times P_y \times P_z$ grid points in a 3D volume of size $N_x \times N_y \times N_z$; equations for all built-in surfaces are reported in Table 1. The meshing is executed via the Matlab built-in function *isosurface,* that discretizes the minimal surfaces into many triangular facets, thus

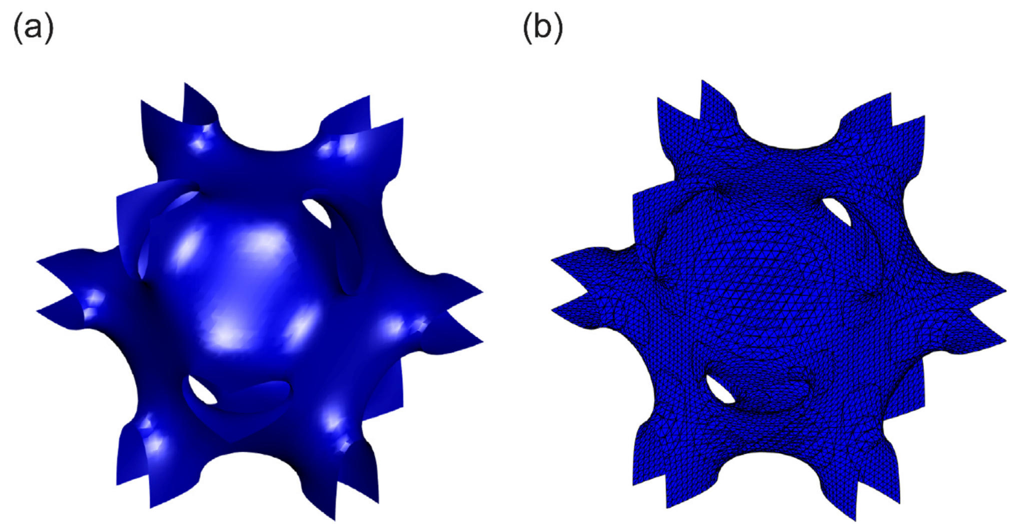

Fig. 3. Display of (a) non-mesh mode and (b) mesh mode of Neovius surface.

Table 1
The level-set surface equations in the form of $f(x, y, z) = c$ for the 19 built-in triply periodic minimal surfaces. For a single unit cell, $x, y,$ and z are bounded by $[0, 2\pi]$.

TPMS	Level-set equation for the TPMS $f(x, y, z) = c$
Schwarz P [30,31]	$\cos(x) + \cos(y) + \cos(z) = 0$
Double Primitive [32]	$0.5\,[\cos(x)\cos(y) + \cos(y)\cos(z) + \cos(z)\cos(x)] + 0.2[\cos(2x) + \cos(2y) + \cos(2z)] = 0$
Schwarz D [31,33]	$\sin(x)\sin(y)\sin(z) + \sin(x)\cos(y)\cos(z) + \cos(x)\sin(y)\cos(z) + \cos(x)\cos(y)\sin(z) = 0$
Complementary D [33]	$\cos(3x + y)\cos(z) - \sin(3x - y)\sin(z) + \cos(x + 3y)\cos(z) + \sin(x - 3y)\sin(z) + \cos(x - y)\cos(3z) - \sin(x + y)\sin(3z) = 0$
Double Diamond [30]	$0.5\,[\sin(x)\sin(y) + \sin(y)\sin(z) + \sin(x)\sin(z)] + 0.5\cos(x)\cos(y)\cos(z) = 0$
D' [30]	$0.5\,[\cos(x)\cos(y)\cos(z) + \cos(x)\sin(y)\sin(z) + \sin(x)\cos(y)\sin(z) + \sin(x)\sin(y)\cos(z)] - 0.5\,[\sin(2x)\sin(2y) + \sin(2y)\sin(2z) + \sin(2z)\sin(2x)] = 0.2$
Gyroid [30,31,33]	$\cos(x)\sin(y) + \cos(y)\sin(z) + \cos(z)\sin(x) = 0$
G' [30]	$\sin(2x)\cos(y)\sin(z) + \sin(2y)\cos(z)\sin(x) + \sin(2z)\cos(x)\sin(y) = -0.32$
Double gyroid [32]	$2.75\,[\sin(2x)\sin(z)\cos(y) + \sin(2y)\sin(x)\cos(z) + \sin(2z)\sin(y)\cos(x)] - [\cos(2x)\cos(2y) + \cos(2y)\cos(2z) + \cos(2z)\cos(2x)] = 0.95$
Karcher K [30]	$0.3\,[\cos(x) + \cos(y) + \cos(z)] + 0.3\,[\cos(x)\cos(y) + \cos(y)\cos(z) + \cos(z)\cos(x)] - 0.4\,[\cos(2x) + \cos(2y) + \cos(2z)] = -0.2$
O, CT-O [30]	$0.6\,[\cos(x)\cos(y) + \cos(y)\cos(z) + \cos(z)\cos(x)] - 0.4\,[\cos(x) + \cos(y) + \cos(z)] = -0.25$
Lidinoid [33,34]	$0.5\,[\sin(2x)\cos(y)\sin(z) + \sin(2y)\cos(z)\sin(x) + \sin(2z)\cos(x)\sin(y)] - 0.5\,[\cos(2x)\cos(2y) + \cos(2y)\cos(2z) + \cos(2z)\cos(2x)] = -0.15$
Neovius [30,31]	$3\,[\cos(x) + \cos(y) + \cos(z)] + 4\cos(x)\cos(y)\cos(z) + \cos(z)\cos(x) = 0$
I-WP [31,33]	$2\,[\cos(x)\cos(y) + \cos(y)\cos(z) + \cos(z)\cos(x)] - [\cos(2x) + \cos(2y) + \cos(2z)] = 0$
Fisher–Koch S [32,33]	$\cos(2x)\sin(y)\cos(z) + \cos(x)\cos(2y)\sin(z) + \sin(x)\cos(y)\cos(2z) = 0$
Fisher–Koch C(S) [33]	$\cos(2x) + \cos(2y) + \cos(2z) + 2\,[\sin(3x)\sin(2y)\cos(z) + \cos(x)\sin(3y)\sin(2z) + \sin(2x)\cos(y)\sin(3z)] +$ $2\,[\sin(2x)\cos(3y)\sin(z) + \sin(x)\sin(2y)\cos(3z) + \cos(3x)\sin(y)\sin(2z)] = 0$
Fisher–Koch Y [33]	$\cos(x)\cos(y)\cos(z) + \sin(x)\sin(y)\sin(z) + \sin(2x)\sin(y) + \sin(2y)\sin(z) + \sin(x)\sin(2z) + \sin(2x)\cos(z) + \cos(x)\sin(2y) + \cos(y)\sin(2z) = 0$
Fisher–Koch C(Y) [33]	$-\sin(x)\sin(y)\sin(z) + \sin(2x)\sin(y) + \sin(2y)\sin(z) + \sin(x)\sin(2z) - \cos(x)\cos(y)\cos(z) + \sin(2x)\cos(z) + \cos(x)\sin(2y) + \cos(y)\sin(2z) = 0$
F-RD [30,32,33]	$4\cos(x)\cos(y)\cos(z) - [\cos(2x)\cos(2y) + \cos(2x)\cos(2z) + \cos(2y)\cos(2z)] = 0$

providing information on the facet-vertex connectivity. Information on the connectivity is then subsequently used to write CAD files in .inp and .stl formats.

3. Impact overview

The idea of using level-set surface equations to approximate TPMSs has been explored extensively in various multidisciplinary research projects for years [35–40]; however, to the best of our knowledge, there is no available software package like *MiniSurf* that includes a nearly complete library of equations for the most interesting TPMSs (with additional ones frequently added) and automatically creates CAD files for both AM and FEM. TPMS shell-based architected materials have remarkable mechanical properties, which makes them superior to classic truss-based lattices in terms of specific strength and toughness [41–43]. These studies are recent and the interest of the mechanics community in the structural performance of TPMS-based materials is only expected to grow. *MiniSurf* will certainly support a number of future projects in this field. As examples, *MiniSurf* is currently used in two ongoing projects in our research group: *(i) Mechanical properties of 3D printed interpenetrating phase composites with shell-based reinforcements* [44]. *MiniSurf* is used to generate CAD files for Schwarz P surface shell-based reinforcements for interpenetrating phase composites. These composites can be readily fabricated by multi-material jetting in VeroWhite (a hard polymeric material for reinforcement) and Agilus (a soft elastomeric material for the matrix) using a Connex 3D printer. The effect of the matrix/reinforcement interpenetration on the mechanical properties of the composites are subsequently investigated both experimentally and numerically (for the numerical studies, *MiniSurf*-generated meshes are used in finite elements analyses of deformation and damage of the composites). *(b) Architected materials designs for long bone implants* [45]. In this effort, we are investigating the performance of minimal surface-based porous materials as implants for long bone repair. Schwarz P CAD files are generated using *MiniSurf* for the purpose of surface area calculations and finite element modeling. The results are then used to draw comparisons among different topological designs and identify optimal topologies.

At the same time, we expect *MiniSurf* to have a broad impact on multidisciplinary studies far beyond the solid mechanics field. The interest of the engineering community in TPMS shell-based materials is documented in several recent studies where TPMS-based architected materials are manufactured and investigated for their multifunctionality, including (1) thermal properties (e.g., thermal conductivity [46,47], coefficient of thermal expansion [48] and heat exchange [49–51]), (2) acoustic properties (e.g., sound absorption and acoustic bandgaps [52,53] and audible coloration [54]) and (3) electrochemical properties (e.g., electrical conductivity [55,56]).

4. Limitations

Despite being user friendly and freely accessible to all researchers and engineers, *MiniSurf* has three main limitations:

(1) *Suboptimal mesh*
In general, meshing in Matlab is done through the Delaunay triangulation algorithm [57,58], which connects a given set of discrete points. Although such algorithm tends to avoid triangular facets with acute angles, meshing of highly curved minimal surfaces – based on the initial user-defined 3D uniform grid points – still results in many triangular facets with bad aspect ratios (thin and long).

(2) *Zero-thickness surface*
MiniSurf generates minimal surfaces composed of many facets without any physical thickness. Postprocessing to thicken these surfaces is often required. Fortunately, many commercial finite element packages (for example, Simulia Abaqus) or additive manufacturing software (for example, Geomagic Design X) have such postprocessing ability.

(3) *Nonparallel computing*
Currently, *MiniSurf* can only execute calculations with one single-core processor, although it can still efficiently generate highly meshed surfaces (300 × 300 × 300 initial mesh grid points) under one minute.

5. Conclusion and future improvements

In this paper, we presented a software package, *MiniSurf*, that efficiently produces CAD files of shell-based architected materials consisting of periodic arrays of minimal surface unit cells, for additive manufacturing and finite element modeling. The surface description is provided via implicit level-set equations. Currently, the software library has 19 built-in minimal surfaces, but any user-defined level-set surface is also allowed. Despite the limitations discussed in Section 4, we expect the software package to be impactful, given the profound interest in TPMS-based architected materials across a wide range of multidisciplinary fields.

In the future, we plan to further improve *MiniSurf* by focusing on its remeshing algorithm, its thickening functionality (triangular facets to triangular prisms), and a parallel computing implementation. Furthermore, we will keep adding new minimal surfaces to our existing library.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] H.N. Wadley, Multifunctional periodic cellular metals, Philos. Trans. R. Soc. A Math. Phys. Eng. Sci. 364 (2006) 31–68, http://dx.doi.org/10.1098/rsta.2005.1697.

[2] Y.H. Zhang, X.M. Qiu, D.N. Fang, Mechanical properties of two novel planar lattice structures, Int. J. Solids Struct. 45 (2008) 3751–3768, http://dx.doi.org/10.1016/j.ijsolstr.2007.10.005.

[3] A. Asadpoure, L. Valdevit, Topology optimization of lightweight periodic lattices under simultaneous compressive and shear stiffness constraints, Int. J. Solids Struct. 60 (2015) 1–16, http://dx.doi.org/10.1016/j.ijsolstr.2015.01.016.

[4] B.P. Russell, V.S. Deshpande, H.N.G. Wadley, Quasistatic deformation and failure modes of composite square honeycombs, 3, 2008.

[5] K.P. Dharmasena, H.N.G. Wadley, Z. Xue, J.W. Hutchinson, Mechanical response of metallic honeycomb sandwich panel structures to high-intensity dynamic loading, Int. J. Impact Eng. 35 (2008) 1063–1074, http://dx.doi.org/10.1016/j.ijimpeng.2007.06.008.

[6] I. Christodoulou, P.J. Tan, Crack initiation and fracture toughness of random Voronoi honeycombs, Eng. Fract. Mech. 104 (2013) 140–161, http://dx.doi.org/10.1016/j.engfracmech.2013.03.017.

[7] M.-T. Hsieh, V.S. Deshpande, L. Valdevit, A versatile numerical approach for calculating the fracture toughness and R-curves of cellular materials, J. Mech. Phys. Solids 138 (2020) 103925, http://dx.doi.org/10.1016/j.jmps.2020.103925.

[8] V.S. Deshpande, N.A. Fleck, M.F. Ashby, Effective properties of the octet-truss lattice material, J. Mech. Phys. Solids 49 (2001) 1747–1769, http://dx.doi.org/10.1016/S0022-5096(01)00010-2.

[9] M.R. O'Masta, L. Dong, L. St-Pierre, H.N.G. Wadley, V.S. Deshpande, The fracture toughness of octet-truss lattices, J. Mech. Phys. Solids 98 (2017) 271–289, http://dx.doi.org/10.1016/j.jmps.2016.09.009.

[10] L. Dong, V. Deshpande, H. Wadley, Mechanical response of ti-6al-4v octet-truss lattice structures, Int. J. Solids Struct. 60 (2015) 107–124, http://dx.doi.org/10.1016/j.ijsolstr.2015.02.020.

[11] X. Wendy Gu, J.R. Greer, Ultra-strong architected Cu meso-lattices, Extrem. Mech. Lett. 2 (2015) 7–14, http://dx.doi.org/10.1016/j.eml.2015.01.006.

[12] A. Bagheri, I. Buj-Corral, M. Ferrer, M.M. Pastor, F. Roure, Determination of the elasticity modulus of 3D printed octet-truss structures for use in porous prosthesis implants, Materials (Basel) 11 (2018) http://dx.doi.org/10.3390/ma11122420.

[13] X.Y. Zheng, H. Lee, T.H. Weisgraber, M. Shusteff, J. Deotte, E.B. Duoss, J.D. Kuntz, M.M. Biener, Q. Ge, J.A. Jackson, S.O. Kucheyev, N.X. Fang, C.M. Spadaccini, Ultralight, ultrastiff mechanical metamaterials, Science 344 (2014) 1373–1378, http://dx.doi.org/10.1126/science.1252291.

[14] L.R. Meza, S. Das, J.R. Greer, Strong, lightweight, and recoverable three-dimensional, Science 345 (2014) 1322–1326, http://dx.doi.org/10.1126/science.1255908.

[15] I. Maskery, L. Sturm, A.O. Aremu, A. Panesar, C.B. Williams, C.J. Tuck, R.D. Wildman, I.A. Ashcroft, R.J.M. Hague, Insights into the mechanical properties of several triply periodic minimal surface lattice structures made by polymer additive manufacturing, Polymer (Guildf) 152 (2018) 62–71, http://dx.doi.org/10.1016/j.polymer.2017.11.049.

[16] O. Al-ketan, R. Rezgui, R. Rowshan, H. Du, N.X. Fang, Microarchitected stretching-dominated mechanical metamaterials with minimal surface, Topologies 1800029 (2018) 1–15, http://dx.doi.org/10.1002/adem.201800029.

[17] M. Helou, S. Kara, Design, Analysis and manufacturing of lattice structures: An overview, Int. J. Comput. Integr. Manuf. 31 (2018) 243–261, http://dx.doi.org/10.1080/0951192X.2017.1407456.

[18] I. Maskery, N.T. Aboulkhair, A.O. Aremu, C.J. Tuck, I.A. Ashcroft, Compressive failure modes and energy absorption in additively manufactured double gyroid lattices, Addit. Manuf. (2017) http://dx.doi.org/10.1016/j.addma.2017.04.003.

[19] L. Bai, C. Gong, X. Chen, Y. Sun, J. Zhang, L. Cai, S. Zhu, S.Q. Xie, Additive manufacturing of customized metallic orthopedic implants: Materials, structures, and surface modifications, Metals (Basel) 9 (2019) http://dx.doi.org/10.3390/met9091004.

[20] A. Ataee, Y. Li, M. Brandt, C. Wen, Ultrahigh-strength titanium gyroid scaffolds manufactured by selective laser melting (SLM) for bone implant applications, Acta Mater. 158 (2018) 354–368, http://dx.doi.org/10.1016/j.actamat.2018.08.005.

[21] M.-T. Hsieh, B. Endo, Y. Zhang, J. Bauer, L. Valdevit, The mechanical response of cellular materials with spinodal topologies, J. Mech. Phys. Solids 125 (2019) 401–419, http://dx.doi.org/10.1016/J.JMPS.2019.01.002.

[22] A. Guell Izard, J. Bauer, C. Crook, V. Turlo, L. Valdevit, Ultrahigh energy absorption multifunctional spinodal nanoarchitectures, Small 466 (2019) 1903834–1903838.

[23] D.M. Kochmann, J.B. Hopkins, L. Valdevit, Multiscale modeling and optimization of the mechanics of hierarchical metamaterials, MRS Bull. 44 (2019) 773–781, http://dx.doi.org/10.1557/mrs.2019.228.

[24] R. Schwaiger, L.R. Meza, X. Li, The extreme mechanics of micro- and nanoarchitected materials, MRS Bull. 44 (2019) 758–765, http://dx.doi.org/10.1557/mrs.2019.230.

[25] S.C. Han, J.W. Lee, K. Kang, A new type of low density material: Shellular, Adv. Mater. 27 (2015) 5506–5511, http://dx.doi.org/10.1002/adma.201501546.

[26] A.E. Garcia, C.S. Wang, R.N. Sanderson, K.M. McDevitt, Y. Zhang, L. Valdevit, D.R. Mumm, A. Mohraz, R. Ragan, Scalable synthesis of gyroid-inspired freestanding three-dimensional graphene architectures, Nanoscale Adv. 350 (2019) 1508–1513.

[27] K.M. McDevitt, T.J. Thorson, E.L. Botvinick, D.R. Mumm, A. Mohraz, Microstructural characteristics of bijel-templated porous materials, Materialia 7 (2019) 100393, http://dx.doi.org/10.1016/j.mtla.2019.100393.

[28] A. Mohraz, T.J. Thorson, Post-processing bijels for applications, in: Bijels Bicontinuous Part. Emuls, The Royal Society of Chemistry, 2020, pp. 34–60, (Chapter 2). http://dx.doi.org/10.1039/9781839160974-00034.

[29] T.J. Thorson, E.L. Botvinick, A. Mohraz, Composite bijel-templated hydrogels for cell delivery, ACS Biomater. Sci. Eng. 4 (2018) 587–594, http://dx.doi.org/10.1021/acsbiomaterials.7b00809.

[30] M. Wohlgemuth, N. Yufa, J. Hoffman, E.L. Thomas, Triply periodic bicontinuous cubic microdomain morphologies by symmetries, Macromolecules 34 (2001) 6083–6089, http://dx.doi.org/10.1021/ma0019499.

[31] A.H. Schoen, Infinite Periodic Minimal Surfaces Without Self-Intersections, 1970.

[32] S.B.G. Blanquer, M. Werner, M. Hannula, S. Sharifi, G.P.R. Lajoinie, D. Eglin, J. Hyttinen, A.A. Poot, D.W. Grijpma, Surface curvature in triply-periodic minimal surface architectures as a distinct design parameter in preparing advanced tissue engineering scaffolds, Biofabrication 9 (2017) http://dx.doi.org/10.1088/1758-5090/aa6553.

[33] K. Michielsen, S. Kole, Photonic band gaps in materials with triply periodic surfaces and related tubular structures, Phys. Rev. B 68 (2003) 1–13, http://dx.doi.org/10.1103/PhysRevB.68.115107.

[34] S. Lidin, S. Larsson, Bonnet transformation of infinite periodic minimal surfaces with hexagonal symmetry, J. Chem. Soc. Faraday Trans. 86 (1990) 769, http://dx.doi.org/10.1039/ft9908600769.

[35] P.J.F. Gandy, S. Bardhan, A.L. Mackay, J. Klinowski, Nodal surface approximations to the P, G, D and I-WP triply periodic minimal surfaces, Chem. Phys. Lett. 336 (2001) 187–195, http://dx.doi.org/10.1016/S0009-2614(00)01418-4.

[36] Y. Jung, S. Torquato, Fluid permeabilities of triply periodic minimal surfaces, Phys. Rev. E 72 (2005) 1–8, http://dx.doi.org/10.1103/PhysRevE.72.056319.

[37] S. Torquato, A. Donev, Minimal surfaces and multifunctionality, Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci. 460 (2004) 1849–1856, http://dx.doi.org/10.1098/rspa.2003.1269.

[38] J.A. Dolan, B.D. Wilts, S. Vignolini, J.J. Baumberg, U. Steiner, T.D. Wilkinson, Optical properties of gyroid structured materials: From photonic crystals to metamaterials, Adv. Opt. Mater. 3 (2015) 12–32, http://dx.doi.org/10.1002/adom.201400333.

[39] B.D. Wilts, K. Michielsen, H. De Raedt, D.G. Stavenga, Iridescence and spectral filtering of the gyroid-type photonic crystals in parides sesostris wing scales, Interface Focus 2 (2012) 681–687, http://dx.doi.org/10.1098/rsfs.2011.0082.

[40] M. Maldovan, A.M. Urbas, N. Yufa, W.C. Carter, E.L. Thomas, Photonic properties of bicontinuous cubic microphases, Phys. Rev. B 65 (2002) 1–5, http://dx.doi.org/10.1103/PhysRevB.65.165123.

[41] S. Rajagopalan, R.A. Robb, Schwarz meets schwann: Design and fabrication of biomorphic and durataxic tissue engineering scaffolds, Med. Image Anal. 10 (2006) 693–712, http://dx.doi.org/10.1016/j.media.2006.06.001.

[42] O. Al-Ketan, R.K.A. Al-Rub, R. Rowshan, Mechanical properties of a new type of architected interpenetrating phase composite materials, Adv. Mater. Technol. 2 (2017) 1–7, http://dx.doi.org/10.1002/admt.201600235.

[43] C. Bonatti, D. Mohr, Mechanical performance of additively-manufactured anisotropic and isotropic smooth shell-lattice materials: Simulations & experiments, J. Mech. Phys. Solids 122 (2019) 1–26, http://dx.doi.org/10.1016/j.jmps.2018.08.022.

[44] Y. Zhang, M.-T. Hsieh, L. Valdevit, Mechanical performance of 3D printed interpenetrating phase composites with spinodal topologies, 2020.

[45] M.-T. Hsieh, M. Begley, L. Valdevit, Architected implant designs for long bones: The advantage of minimal surface-based topologies, Acta Biomaterialia, in preparation.

[46] D.W. Abueidda, R.K. Abu Al-Rub, A.S. Dalaq, D.W. Lee, K.A. Khan, I. Jasiuk, Effective conductivities and elastic moduli of novel foams with triply periodic minimal surfaces, Mech. Mater. 95 (2016) 102–115, http://dx.doi.org/10.1016/j.mechmat.2016.01.004.

[47] G.S. Jung, J. Yeo, Z. Tian, Z. Qin, M.J. Buehler, Unusually low and density-insensitive thermal conductivity of three-dimensional gyroid graphene, Nanoscale 9 (2017) 13477–13484, http://dx.doi.org/10.1039/c7nr04455k.

[48] D.W. Abueidda, A.S. Dalaq, R.K. Abu Al-Rub, I. Jasiuk, Micromechanical finite element predictions of a reduced coefficient of thermal expansion for 3D periodic architectured interpenetrating phase composites, Compos. Struct. 133 (2015) 85–97, http://dx.doi.org/10.1016/j.compstruct.2015.06.082.

[49] H. Peng, F. Gao, W. Hu, Design, modeling and characterization of triply periodic minimal surface heat exchangers with additive manufacturing, 2019, pp. 2325–2337.

[50] J. Kim, D.-J. Yoo, 3D Printed compact heat exchangers with mathematically defined core structures, J. Comput. Des. Eng. 7 (2020) 1–24, http://dx.doi.org/10.1093/jcde/qwaa032.

[51] W. Li, G. Yu, Z. Yu, Bioinspired heat exchangers based on triply periodic minimal surfaces for supercritical CO2 cycles, Appl. Therm. Eng. 179 (2020) 115686, http://dx.doi.org/10.1016/j.applthermaleng.2020.115686.

[52] W. Yang, J. An, C.K. Chua, K. Zhou, Acoustic absorptions of multifunctional polymeric cellular structures based on triply periodic minimal surfaces fabricated by stereolithography, Virtual Phys. Prototyp. 15 (2020) 242–249, http://dx.doi.org/10.1080/17452759.2020.1740747.

[53] D.W. Abueidda, I. Jasiuk, N.A. Sobh, Acoustic band gaps and elastic stiffness of PMMA cellular solids based on triply periodic minimal surfaces, Mater. Des. 145 (2018) 20–27, http://dx.doi.org/10.1016/j.matdes.2018.02.032.

[54] D.S.H. Murphy, Simulation of acoustic wave propagation in 3-D sonic crystals based on triply periodic minimal surfaces, Joimt Balt. Acoust. Meet. (2012).

[55] A.A.-R. R.K., A. D.W., D. A.S., Thermo-electro-mechanical properties of interpenetrating phase composites with periodic architectured reinforcements, in: A. H., M. T., O. D. (Eds.), From Creep Damage Mech. To Homog. Methods, Springer, Cham, 2015, http://dx.doi.org/10.1007/978-3-319-19440-0_1.

[56] X.C. Ye, X.C. Lin, J.Y. Xiong, H.H. Wu, G.W. Zhao, D. Fang, Electrical properties of 3D printed graphite cellular lattice structures with triply periodic minimal surface architectures, Mater. Res. Express. 6 (2019) http://dx.doi.org/10.1088/2053-1591/ab569b.

[57] C.B. Barber, D.P. Dobkin, H. Huhdanpaa, The quickhull algorithm for convex hulls, ACM Trans. Math. Software 22 (1996) 469–483, http://dx.doi.org/10.1145/235815.235821.

[58] D.T. Lee, B.J. Schachter, Two algorithms for constructing a delaunay triangulation, Int. J. Comput. Inf. Sci. 9 (1980) 219–242, http://dx.doi.org/10.1007/BF00977785.